Exercise 10.2B Reasoning and problem-solving

1 a In a repeated set of trials, X is a random variable for the total number of 'successes'. State three conditions required to be able to model X by a binomial distribution.

In 2013, the proportion of households purchasing milk that was fully- or semi-skimmed was 7.0%. An independent random sample of size 200 households was taken.

b Find the probability that **i** Exactly 16 **ii** Fewer than 16 **iii** Between 10 and 16 (inclusive) households were purchasing fully- or semi-skimmed milk.

2 For each of the following random variables, state whether the binomial distribution can be used as a good probability model. If it can, state the values of n and p; if it can't, or if its use is questionable, give reasons.

a The number of black counters obtained when 4 counters are chosen, with each being returned before the next is chosen, from a bag containing 6 black and 8 white counters.

b The number of patients in an independent random sample of size 8 at a GP practice who are prescribed antibiotics. You are given that 12% of patients are prescribed antibiotics.

c The number of heads in 5 throws of a biased coin where the probability of a head is 0.6

d The number of throws of a fair coin up to and including the first head.

3 A calculator claims it can randomly generate a digit from 0–9. For any 4 digits generated, the probability of 2 zeros is 0.03. Is the calculator's claim correct? Show your working.

4 In 2012, half of all households in England purchased over 66 g worth of filled chocolate bars per person per week.

a In a random sample of 40 households, find the probability that **i** 23 **ii** Fewer than 18 households purchased more than 66 g worth of filled chocolate bars per person per week.

b Write down the probability that, of the 40 households **i** More than 17 **ii** More than 22 purchased more than 66 g worth of filled chocolate bars per person per week.

5 The quantity of milk and milk products (in ml) purchased per person per week was recorded in 2007. It was found that, in England, 41% of people purchased less than 1944 ml per week. In 2013, this proportion had risen to 50%.

a In a random sample of 70 people taken in 2007, find the probability that more than 30 purchased less than 1944 ml of milk and milk products per week.

b How would the probability in part **a** change if the same investigation had taken place in 2013. Give a reason for your answer.

6 A pair of fair six-sided dice is thrown eight times. Find the probability that a score greater than 7 is scored no more than five times.

7 Somebody claims they can tell the difference between two different brands, A and B, of tea. They are given 5 pairs of cups, where in each pair 1 cup contains brand A and 1 contains brand B. Assuming that they are guessing, find the probability that they correctly identify at least 3 pairs.

Challenge

8 For any family of 5 children, A is the event 'there is at least 1 boy and 1 girl' and B is the event 'there are more girls than boys'. A symmetrical binomial probability distribution can model X, the number of girls in a family of 5 children.

Are the events A and B independent of each other? Show your working.

Exercise 10.2B Reasoning and problem solving

1a There should be a fixed number of independent and identical trials.

1b X = number of households using milk that was fully- or semi-skimmed.

$X \sim B(200, 0.07)$.

i 0.089

ii 0.673

iii $P(10 \le X \le 16) = P(X \le 16) - P(X \le 9)$

$= 0.661$ (3 dp)

2a Yes. $n = 4$, $p = \frac{6}{14} = \frac{3}{7}$

2b Yes. If the population is sufficiently large when compared to the sample, as the patients are chosen at random the probability of getting a patient who will be prescribed antibiotics remains constant.

$n = 8$, $p = 0.12$

2c Yes.

$n = 5$, $p = 0.6$

2d No.

Number of trials not fixed.

3 No.

If it was, the probability of two zeros would be 0.0486

4 X = number spending more than £66.

$X \sim B(40, 0.5)$

a **i** $P(X = 23) = 0.081$ (to 3 dp)

ii $P(X < 18) = P(X \le 17)$

$= 0.215$ (to 3 dp)

b **i** $1 - P(X \le 17) = 0.785$ (to 3 dp)

ii $P(X > 22) = P(X \ge 23)$

$p = 0.5$ so the distribution is symmetrical.

$P(X \ge 23) = P(X \le 17)$

$= 0.215$ (to 3 dp)

5 X = number out of 70 with energy intake less than 1944 kcals.

a $X \sim B(70, 0.41)$

$P(X > 30) = 1 - P(X \le 30)$

$= 0.329$

b Increase. The probability of the event 'an energy intake of less than 1944 kcals' is higher so the probability of X taking a high value would be higher.

6

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Let X be the number of times the score is greater than seven in eight throws.

$P(\text{score more than } 7) = \frac{15}{36} = \frac{5}{12}$

So $X \sim B\left(8, \frac{5}{12}\right)$

$P(X \le 5) = 0.939$ (3 sf)

7 Let X be a random variable for the number of correct identifications in five pairs.

If guessing, $P(\text{correct}) = 0.5$

$X \sim B(5, 0.5)$

$P(X \ge 3) = 1 - P(X \le 2) = 0.5$

8 Let X be a random variable for the number of girls born in five children.

$\mathrm{P(girl)} = 0.5$ so $X \sim \mathrm{B}(5, 0.5)$

$$\begin{aligned}
\mathrm{P}(A) &= \mathrm{P}(X \text{ is not } 0 \text{ or } 5) \\
&= 1 - \mathrm{P}(X = 0) - \mathrm{P}(X = 5) \\
&= 1 - 0.03125 - 0.03125 \\
&= 0.9375
\end{aligned}$$

$$\begin{aligned}
\mathrm{P}(B) &= \mathrm{P}(X \geq 3) \\
&= 1 - \mathrm{P}(X \leq 2) \\
&= 1 - 0.5 \\
&= 0.5
\end{aligned}$$

$$\begin{aligned}
\mathrm{P}(A \text{ and } B) &= \mathrm{P}(X = 3 \text{ or } 4) \\
&= \mathrm{P}(X = 3) + \mathrm{P}(X = 4) \text{ as outcomes mutually exclusive} \\
&= 0.3125 + 0.15625 \\
&= 0.46875
\end{aligned}$$

$$\begin{aligned}
\mathrm{P}(A) \times \mathrm{P}(B) &= 0.9375 \times 0.5 \\
&= 0.46875 \\
&= \mathrm{P}(A \text{ and } B)
\end{aligned}$$

So events are independent.